Using the Proper Generalized Decomposition to solve Maxwell equations in thin laminated composites

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***Abstract* — Microwave heating relies on internal thermal sources which make it a good candidate for an alternate composite process. In this paper, an electromagnetic model is proposed to simulate the electromagnetic wave interaction with the material. Solving the problem in a laminated composite material requires a high degree of discretization in the thickness direction which is made possible by introducing the in-plane-out-of-plane decomposition approach using the Proper Generalized Decomposition (PGD).**

I. INTRODUCTION

Composite parts use is spread throughout the transport industry (automotive and aeronautic) thanks to their combination of high mechanical properties and low mass. However, their long cycle time is still a disadvantage.

Conversely to the conventional processing methods which rely on surface heat transfer, microwave (MW) technology depends on volumetric heating. It enables better process temperature control [1] and less overall energy use, which can results in shorter processing cycles. The energy efficiency of the process is maximized as the volumetric heating occurs directly in the part to be processed, and no energy is wasted by heating the surrounding air or tool. Substantial time and energy benefits have been reported [2] and these virtues of the MW technology have attracted interest in developing the method. But the main drawback today is that the complex physics involved in the conversion of electromagnetic (EM) energy to thermal energy is not entirely understood and controlled.

The objective of this work is to model and simulate the propagation of the MW field in a laminated composite material in a 3D solver. The main challenge concerns the need for a high-resolution description of the EM field in such a material which involves plies whose in-plane dimensions are order of magnitude higher than the thickness one (typical aspect of ratio of tens of thousands). In that situation, the use of the in-plane-out-of-plane separated representation within the Proper Generalized Decomposition (PGD) framework is an appealing and valuable route for solving 3D models while having a computational complexity of standard 2D models [3].

II. ELECTROMAGNETIC SOLVER

Here, the usual approach when solving an EM problem, i.e. using edge elements with the double-curl Maxwell equation is not considered. Despite its advantages, drawbacks of this approach have been pointed out [4]. In order to proceed with the in-plane-out-of-plane separated representation we use a standard nodal formulation that is regularized in order to avoid spurious solutions.

A. Electromagnetic formulation

The double-curl formulation is derived from the constitutive equations and the Maxwell equations in the frequency space. In absence of current density, the electric field $\boldsymbol{E}$ equation reads:

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \boldsymbol{E}\right) - \omega^2 \varepsilon^* \boldsymbol{E} = 0 \quad \text{with} \quad \varepsilon^* = \varepsilon - i\frac{\sigma}{\omega} \qquad (1)$$

where μ, ε and σ are respectively the permeability, the permittivity and the conductivity of the material. (1) is complemented with adequate Dirichlet boundary conditions in the whole domain boundary $\partial\Omega$

$$\boldsymbol{n} \times \boldsymbol{E} = \boldsymbol{E}^t \qquad (2)$$

where $\boldsymbol{E}^t$ is the prescribed electric field (assumed known) on the domain boundary.

However, it is known that the weak form associated to (1) leads to spurious solutions [5] as the implied discretization in the weak equation prevent from the verification of the Gauss equation $\nabla \cdot (\varepsilon^* \boldsymbol{E}) = 0$ despite the fact that it is implied in the strong formulation. As a consequence, the regularized equation is considered:

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \boldsymbol{E}\right) - \overline{\varepsilon^*} \left(\frac{1}{\mu \overline{\varepsilon^*} \varepsilon^*} \nabla \cdot (\varepsilon^* \boldsymbol{E})\right) - \omega^2 \varepsilon^* \boldsymbol{E} = 0 \qquad (3)$$

The weak formulation associated to (3) is solved in the whole domain Ω with the appropriate Dirichlet boundary conditions.

B. In-plane-out-of-plane separated representation

As mentioned before, the work is carried using the in-plane-out-of-plane separated representation within the Proper Generalized Decomposition (PGD) which allows writing the electric field in the 3D separated form:

$$\boldsymbol{E}(x, y, z) \approx \sum_{i=1}^{N} \boldsymbol{X}_i(x, y) \circ \boldsymbol{Z}_i(z) \qquad (4)$$

where the bullet $\circ$ denotes the Hadamar product. Thus, the electric field is expressed as a finite sum of functional couples involving a function depending on the in-plane coordinates (x, y) and the other the thickness coordinate (z). Thus, the 3D solution is obtained from N 2D problems involving the in-plane coordinates and the same number of 1D problems involving the thickness.

Therefore, we can reach levels of resolution related to hundreds of degrees of freedom along the thickness direction (that has a characteristic size of few millimeters) without having any impact on the in-plane representation, and then in the computational efficiency. All the details of the solution procedure can be found in [6].

C. Interface conditions

The electric field propagation at interfaces is ruled by the following equation in a free-charge space:

$$(\varepsilon_1 \boldsymbol{E_1} - \varepsilon_2 \boldsymbol{E_2}) \cdot \boldsymbol{n} = 0 \quad (5)$$

$$(\boldsymbol{E_1} - \boldsymbol{E_2}) \times \boldsymbol{n} = 0 \quad (6)$$

These interfaces conditions highlight the continuity of the electric field for the tangential components and the discontinuity of the normal component. In the case we are considering, it is assumed that the only interfaces are along the thickness (z coordinate), thus we use a standard continuous nodal approximation of fields E_x and E_y. For the E_z component, we have to consider the fact that it is continuous in the plane but discontinuous at interfaces where the field jump is:

$$\varepsilon_1 E_z(z^-) = \varepsilon_2 E_z(z^+) \quad (7)$$

where $E_z(z^-)$ and $E_z(z^+)$ are the electric field at both sides of the interface. In order to enforce the discontinuity, the nodes at interfaces in the out-of-plane mesh used to approximate the z-component of $\boldsymbol{Z_i}$ functions, are duplicated, each one being used to approximate the solution at one side of the interface.

III. NUMERICAL RESULTS

We consider a composite part placed in a wave-transparent ceramic closed mold as depicted on Fig. 1.

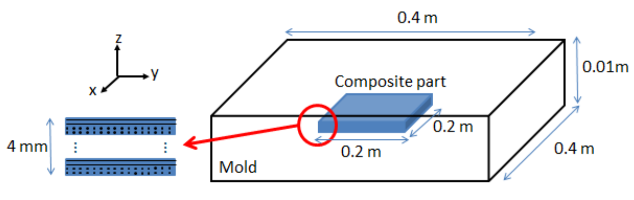

Fig.1. Test case geometry

The composite material is made of 20 unidirectional layers with the following stacking sequence $[(0°/90°)_5]_S$. Material properties of the mold are $\sigma = 0.008$ $\varepsilon = 4\varepsilon_0, \mu = \mu_0$, ε_0 and μ_0 being the permeability and permittivity of the vacuum and the 0°-unidirectional layer is characterised by:

$$\sigma = \begin{pmatrix} 100 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} S/m,\ \varepsilon = \begin{pmatrix} 90 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix} \varepsilon_0 \text{ and}$$

$$\mu = \mu_0 \quad (8)$$

The Dirichlet boundary conditions applied on the whole domain are:

$$\boldsymbol{n} \times \boldsymbol{E} = \begin{pmatrix} \cos(2\pi n x) \\ \cos(2\pi n x) \\ \cos(2\pi n y) \end{pmatrix} \text{ with } n = 10 \quad (9)$$

The mesh is composed of 1000 Q4 elements in the plane and 980 1D linear elements in the thickness.

Results are depicted below. Fig. 2 highlights the electric field absorption as it penetrates in the composite part. Fig. 3 depicts the field discontinuity for the z-component at the interfaces between the mold and the composite part.

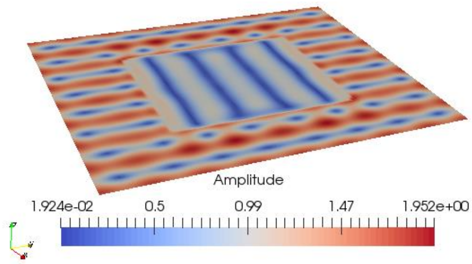

Fig.2. Electric field amplitude in the cross-section (x, y, z_{middle})

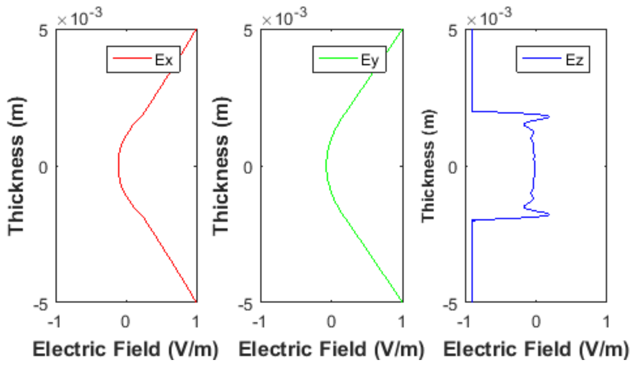

Fig.3. Electric field through the thickness along $(x_{middle}, y_{middle}, z)$

IV. CONCLUSION

The modeling of the electromagnetic waves propagation in a thin laminate composite part has been proposed. Thanks to the simulation approach chosen, the model is able to ensure as refined results as needed and to take into account field discontinuity at interfaces.

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